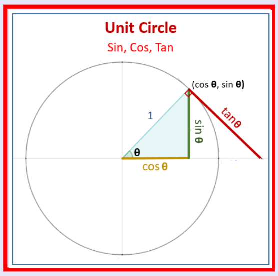
Math 241

Winter 2024

Lecture 10

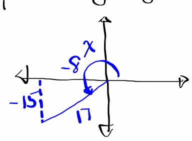


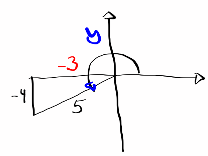
Feb 19-8:47 AM

Given $\cos x = \frac{-8}{17}$, $\sin y = \frac{-4}{5}$, x & y are in QIII.

Find

1) $\sin(x+y)$





$y^2 + (-8)^2 = 17^2$

$y = 15$

2) $\cos(x-y)$

3) $\tan 2x$

$= \tan(x+x) \longrightarrow \tan(A+B)$

$= \dfrac{\tan x + \tan x}{1 - \tan x \cdot \tan x}$

$= \dfrac{2\tan x}{1 - \tan^2 x}$

$= \dfrac{\cancel{2} \cdot \frac{15}{\cancel{8}}}{1 - \left(\frac{15}{8}\right)^2} = \dfrac{\frac{15}{4}}{1 - \frac{225}{64}}$

LCD $= 64$ $\quad = \dfrac{\overset{16}{\cancel{64}} \cdot \frac{15}{\cancel{4}}}{64 \cdot 1 - \cancel{64} \cdot \frac{225}{\cancel{64}}} = \dfrac{240}{64 - 225} = \dfrac{240}{-161}$

$= \boxed{-\dfrac{240}{161}}$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$= \frac{-15}{17} \cdot \frac{-3}{5} + \frac{-8}{17} \cdot \frac{-4}{5}$

$= \dfrac{45 + 32}{85} = \boxed{\dfrac{77}{85}}$

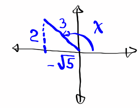
$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$= \frac{-8}{17} \cdot \frac{-3}{5} + \frac{-15}{17} \cdot \frac{-4}{5}$

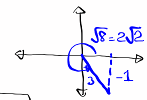
$= \dfrac{24 + 60}{85} = \boxed{\dfrac{84}{85}}$

Jan 18-8:02 AM

$\sin x = \frac{2}{3}$, x is in QII



$\sin y = \frac{-1}{3}$, y is in QIV



Find

1) $\sin 2x = \sin(x + x) \rightarrow \sin(A+B)$

$= \sin x \cos x + \cos x \sin x$

$= 2 \sin x \cos x$

$= 2 \cdot \frac{2}{3} \cdot \frac{-\sqrt{5}}{3} = \boxed{\frac{-4\sqrt{5}}{9}}$

2) $\cos 2x$

$= \cos(x + x) = \cos x \cos x - \sin x \sin x = \cos^2 x - \sin^2 x$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \left(\frac{-\sqrt{5}}{3}\right)^2 - \left(\frac{2}{3}\right)^2 = \frac{5}{9} - \frac{4}{9} = \boxed{\frac{1}{9}}$

3) $\tan(x-y)$

$$= \frac{\tan x - \tan y}{1 + \tan x \tan y} = \frac{\frac{-2}{\sqrt{5}} - \frac{-1}{2\sqrt{2}}}{1 + \frac{-2}{\sqrt{5}} \cdot \frac{-1}{2\sqrt{2}}}$$

$$= \frac{\frac{-2}{\sqrt{5}} + \frac{1}{2\sqrt{2}}}{1 + \frac{2}{\sqrt{5} \cdot 2\sqrt{2}}}$$

LCD $= 2\sqrt{5}\sqrt{2}$

$$= \frac{-4\sqrt{2} + \sqrt{5}}{2\sqrt{5}\sqrt{2} + 2} = \frac{-4\sqrt{2} + \sqrt{5}}{2\sqrt{10} + 2} \cdot \frac{2\sqrt{10} - 2}{2\sqrt{10} - 2}$$

$$= \frac{-8\sqrt{20} + 8\sqrt{2} + 2\sqrt{50} - 2\sqrt{5}}{\underbrace{4\sqrt{100}}_{40} - \cancel{4\sqrt{10}} + \cancel{4\sqrt{10}} - 4}$$

$$= \frac{-16\sqrt{5} + 8\sqrt{2} + 10\sqrt{2} - 2\sqrt{5}}{36} = \frac{18\sqrt{2} - 18\sqrt{5}}{36}$$

$$= \frac{\cancel{18}(\sqrt{2} - \sqrt{5})}{\cancel{36}_{2}} = \boxed{\frac{\sqrt{2} - \sqrt{5}}{2}}$$

Jan 18-8:17 AM

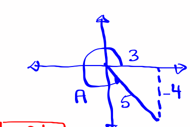
Double − Angle

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\cos A = \frac{3}{5}$, $\sin A < 0$, Find



$\sin 2A = 2 \sin A \cos A = 2 \cdot \frac{-4}{5} \cdot \frac{3}{5} = \boxed{\frac{-24}{25}}$

$\cos 2A = 2\cos^2 A - 1 = 2\left(\frac{3}{5}\right)^2 - 1 = \frac{18}{25} - \frac{25}{25} = \boxed{\frac{-7}{25}}$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \cdot \frac{-4}{3}}{1 - \left(\frac{-4}{3}\right)^2} = \frac{\frac{-8}{3}}{1 - \frac{16}{9}}$$

$$= \frac{\cancel{9}^{3} \cdot \frac{-8}{\cancel{3}}}{9 \cdot 1 - \cancel{9} \cdot \frac{16}{\cancel{9}}} = \frac{-24}{9 - 16} = \frac{-24}{-7} = \boxed{\frac{24}{7}}$$

LCD $= 9$

Jan 18-8:35 AM

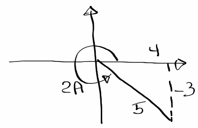
$\cos 2A = \frac{4}{5}$, $90^\circ < A < 180^\circ$

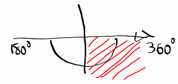
find $\tan 2A$

$2\cdot 90^\circ < 2A < 2\cdot 180^\circ$

$180^\circ < 2A < 360^\circ$

But $\cos 2A > 0$





$\boxed{\tan 2A = \frac{-3}{4}}$ ✓

Sin A , Cos A, and Tan A

\+   −   −

$\cos 2A = 2\cos^2 A - 1$

$\frac{4}{5} = 2\cos^2 A - 1$

$\frac{4}{5} + 1 = 2\cos^2 A$

$\frac{9}{5} = 2\cos^2 A$

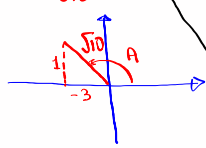
$\cos^2 A = \frac{9}{10}$

$\cos A = \frac{-3}{\sqrt{10}}$

$\sin A = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\cos A = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10}$

$\boxed{\tan A = \frac{-1}{3}}$



$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\cdot\frac{-1}{3}}{1-\frac{1}{9}}$

LCD=9

$= \frac{\cancel{9}^{3}\cdot\frac{-2}{\cancel{3}}}{9\cdot 1 - \cancel{9}\cdot\frac{1}{\cancel{9}}}$

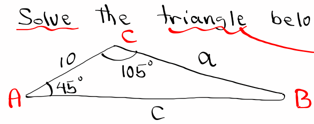
$= \frac{-6}{9-1} = \frac{-6}{8} = \boxed{\frac{-3}{4}}$

A+B, A−B, 2A

for exam II

Jan 18-8:46 AM

Solve the triangle below → find missing sides & angles.



$A + B + C = 180^\circ$

$45^\circ + B + 105^\circ = 180^\circ \rightarrow \boxed{B = 30^\circ}$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 45^\circ} = \frac{10}{\sin 30^\circ} = \frac{c}{\sin 105^\circ}$$

$a \sin 30^\circ = 10\cdot\sin 45^\circ$

$a\cdot\frac{1}{2} = 10\cdot\frac{\sqrt{2}}{2}$

$\boxed{a = 10\sqrt{2}}$

$c\cdot\sin 30^\circ = 10\cdot\sin 105^\circ$

$c\cdot\frac{1}{2} = 10\cdot\sin 105^\circ$

$c = 20\cdot\sin 105^\circ$

$= \cancel{20}^{5}\cdot\frac{\sqrt{6}+\sqrt{2}}{\cancel{4}}$

$= 5(\sqrt{6}+\sqrt{2})$

$\sin 105^\circ = \sin(60^\circ + 45^\circ)$

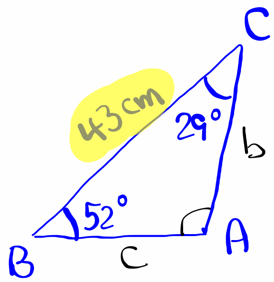
$= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

Jan 18-9:01 AM

Solve the triangle below. Round to whole #.



$A + B + C = 180°$

$A + 52° + 29° = 180° \rightarrow \boxed{A = 99°}$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\boxed{\frac{43}{\sin 99°}} = \frac{b}{\sin 52°} = \boxed{\frac{c}{\sin 29°}}$$

$b \sin 99° = 43 \sin 52°$

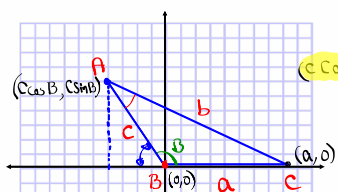
$$b = \frac{43 \sin 52°}{\sin 99°} \approx \boxed{34\text{ cm}}$$

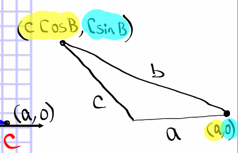
$c \cdot \sin 99° = 43 \cdot \sin 29°$

$$c = \frac{43 \cdot \sin 29°}{\sin 99°}$$

$\boxed{c \approx 21\text{ cm}}$

Jan 18-9:10 AM





$\sin B = \frac{y}{c}$, $\cos B = \frac{x}{c}$

$y = c \sin B$ $\quad x = c \cos B$

Distance formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$b = \sqrt{(c\cos B - a)^2 + (c \sin B - 0)^2}$

$b^2 = (c\cos B - a)^2 + (c \sin B)^2$

$b^2 = c^2\cos^2 B - 2ac\cos B + a^2 + c^2\sin^2 B$

$= c^2\cos^2 B + c^2 \sin^2 B - 2ac\cos B + a^2$

$= c^2(\underbrace{\cos^2 B + \sin^2 B}_{1}) - 2ac\cos B + a^2$

$\boxed{b^2 = a^2 + c^2 - 2ac\cos B}$

$a^2 = b^2 + c^2 - 2bc\cos A$

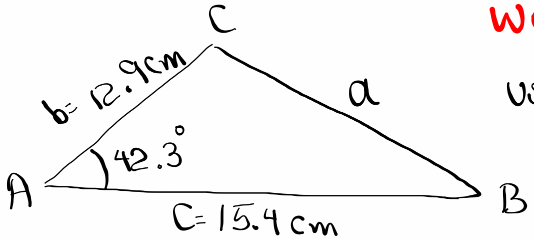
$c^2 = a^2 + b^2 - 2ab\cos C$

Law of Cosines

SAS

Jan 18-9:33 AM

Find a. Round to 1-decimal.



We have SAS

use Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 12.9^2 + 15.4^2 - 2 \cdot 12.9 \cdot 15.4 \cdot \cos 42.3°$

$a^2 = 109.6997733$

$a = \sqrt{109.6997733}$

$\boxed{a \approx 10.5\text{ cm}}$

Jan 18-9:46 AM

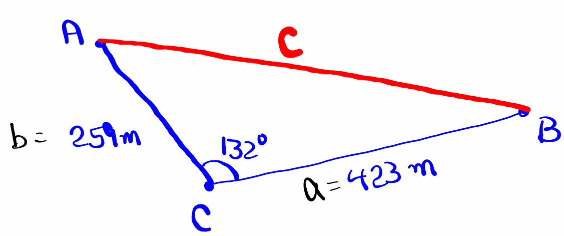
Point C is 259 m from Point A.

Point C is 423 m from Point B

The angle ACB is 132°.

Find the distance from A to B.

Drawing Required.



We have SAS

Law of Cosines

$c^2 = a^2 + b^2 - 2ab\cos C \quad = 423^2 + 259^2 - 2 \cdot 423 \cdot 259 \cdot \cos 132°$

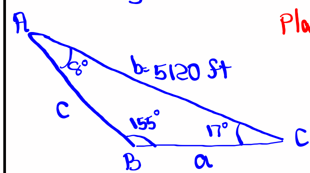
$c^2 = 392625.8837 \longrightarrow \boxed{c \approx 627\text{ m}}$

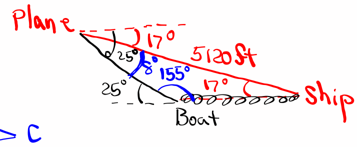
Jan 18-9:51 AM

The angle of depression from a plane to a ship is 17°.
Plane is 5120 ft from the ship.

The angle of depression from the plane to a boat is 25°.

Find the distance between ship & boat.
Drawing Required.





Using Law of Sines $\frac{a}{\sin 8°} = \frac{5120}{\sin 155°}$

$a \sin 155° = 5120 \sin 8°$

$a = \frac{5120 \sin 8°}{\sin 155°} \approx \boxed{1686 \text{ ft}}$

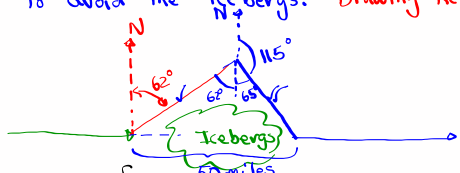
Jan 18-9:59 AM

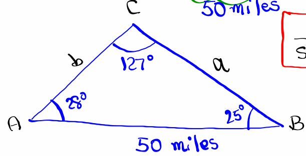
Titanic is heading East.

Captain is changing direction with 62° bearing to avoid iceberg.

Then it changes direction with 115° bearing to reaches its original course

How much further did titanic have to travel to avoid the icebergs? Drawing Required.





$\boxed{\frac{a}{\sin 28°}} = \frac{b}{\sin 25°} = \boxed{\frac{50}{\sin 127°}}$

$a \sin 127° = 50 \cdot \sin 28°$

$a \approx 29.4$ miles

$b \sin 127° = 50 \sin 25°$

$b \approx 26.5$ miles

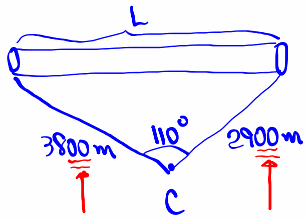
$29.4 + 26.5 - 50$

≈ 5.9 miles

Jan 18-10:10 AM

To measure the length of a tunnel, the Surveyor pick a point and measures to each end of the tunnel.

Suppose the angle between the lines from that point is 110°. Find length of the tunnel if the point is 3800 m and 2900 m from the end of tunnel.



$$L^2 = 2900^2 + 3800^2 - 2 \cdot 2900 \cdot 3800 \cdot \cos 110^\circ$$

$L^2 =$

$\boxed{L \approx 5513 \text{ m}}$

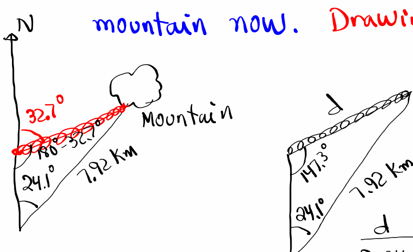
To the nearest 100
5 500 m

Jan 18-10:27 AM

A plane is heading North.

Captain observes a mountain with bearing of 24.1°, and it is 7.92 Km from the mountain.

A short time later, the bearing becomes 32.7°. How far is the plane from the mountain now. Drawing Required



$$\frac{d}{\sin 24.1^\circ} = \frac{7.92}{\sin 147.3^\circ}$$

$\boxed{d \approx 5.99 \text{ Km}}$

Law of Sines
Law of Cosines
Exam II

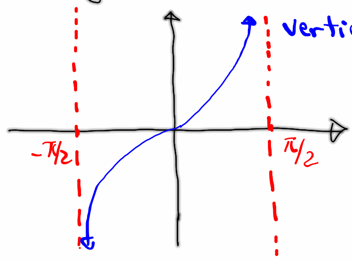
Jan 18-10:35 AM

Graphing $y = \tan x$

Domain $-\frac{\pi}{2} < x < \frac{\pi}{2}$ $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Range $(-\infty, \infty)$

One period of Graph



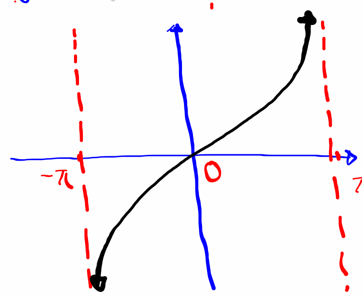
Vertical Asymptote

Recall $\tan x = \frac{\sin x}{\cos x}$ $\cos \frac{\pi}{2} = 0$ $\cos -\frac{\pi}{2} = 0$

Graph $y = \tan \frac{1}{2} x$

$-\frac{\pi}{2} < \frac{1}{2} x < \frac{\pi}{2}$

$-\pi < x < \pi$

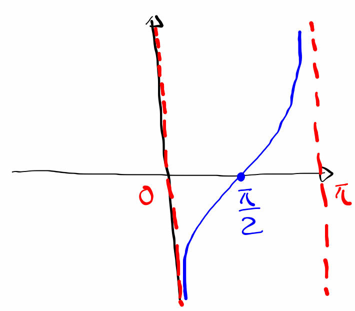


Jan 18-11:04 AM

Graph $y = \tan\left(x - \frac{\pi}{2}\right)$

$-\frac{\pi}{2} < x - \frac{\pi}{2} < \frac{\pi}{2}$

$0 < x < \pi$



Graph $y = \tan\left(2x + \frac{\pi}{2}\right)$

$-\frac{\pi}{2} < 2x + \frac{\pi}{2} < \frac{\pi}{2}$

$-\frac{\pi}{2} - \frac{\pi}{2} < 2x \cancel{+\frac{\pi}{2}} \cancel{-\pi/2} < \frac{\pi}{2} - \frac{\pi}{2}$

$-\pi < 2x < 0$

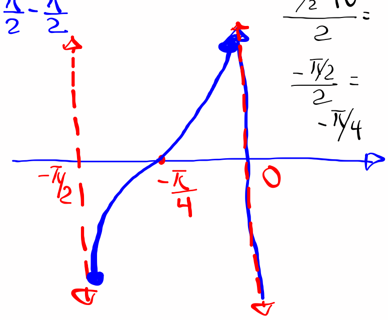
$\frac{-\pi}{2} < x < \frac{0}{2}$

$\frac{-\pi}{2} < x < 0$

$\frac{-\pi/2 + 0}{2} =$

$\frac{-\pi/2}{2} =$

$-\pi/4$



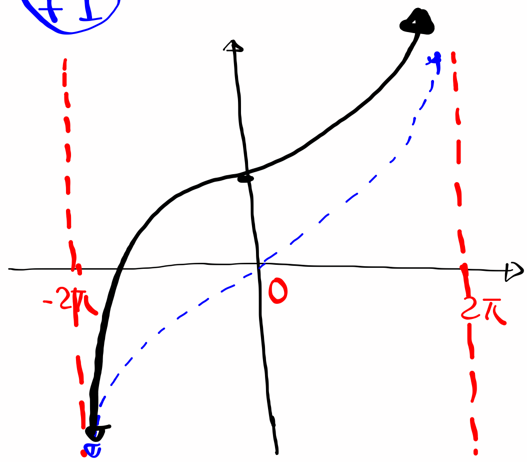
Jan 18-11:09 AM

Graph $y = \tan\left(\frac{1}{4}x\right)$ $+1$

$-\frac{\pi}{2} < \frac{1}{4}x < \frac{\pi}{2}$

LCD = 4

$-2\pi < x < 2\pi$



Jan 18-11:17 AM

Graph $y = \tan(-2x)$

Be careful

$\frac{\pi/4}{2} = \pi/8$

$\tan(-2 \cdot \pi/8) =$

$\tan(-\pi/4) = -1$

Method I:

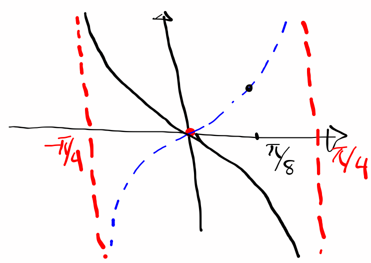
$-\pi/2 < -2x < \pi/2$

$-\pi < -4x < \pi$

$\frac{-\pi}{-4} > \frac{-4}{-4}x > \frac{\pi}{-4}$

$\frac{\pi}{4} > x > -\frac{\pi}{4}$

$-\frac{\pi}{4} < x < \frac{\pi}{4}$



Method II:

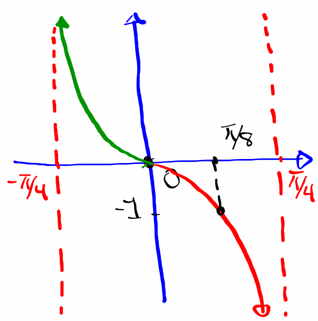
Recall $\tan(-\alpha) = -\tan\alpha$

$\tan(-2x) = -\tan 2x$

$y = \tan(-2x) = -\tan 2x$

$-\pi/2 < 2x < \pi/2$

$-\pi/4 < x < \pi/4$



Jan 18-11:21 AM

Graph $y = -2\tan\left(-\frac{1}{2}x\right) - 4$

Recall $\tan(-\alpha) = -\tan\alpha$

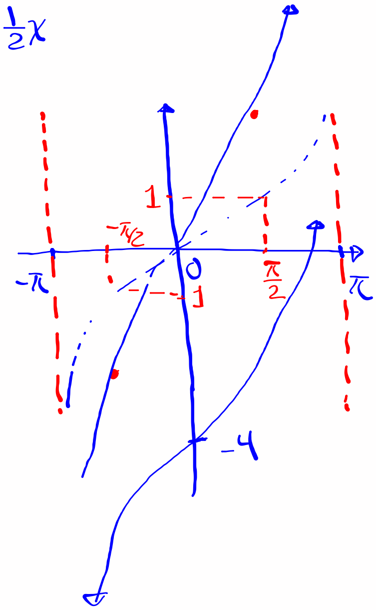
$\tan\left(-\frac{1}{2}x\right) = -\tan\frac{1}{2}x$

$y = -2 \cdot -\tan\frac{1}{2}x - 4$

$y = 2\tan\frac{1}{2}x - 4$

$-\frac{\pi}{2} < \frac{1}{2}x < \frac{\pi}{2}$

$-\pi < x < \pi$



Jan 18-11:33 AM

Graph $y = \sin(-2x)$

Recall

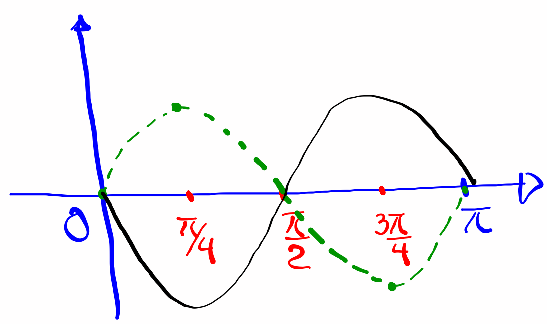
$\sin(-\alpha) = -\sin\alpha$

$\sin(-2x) = -\sin 2x$

Graph $y = -\sin 2x$

$0 \le 2x \le 2\pi$

$0 \le x \le \pi$



Jan 18-11:39 AM

Graph $y = -2\cos(\pi - \frac{1}{2}x) + 2$

$y = \cos(\pi - \frac{1}{2}x)$

Plug in -2π

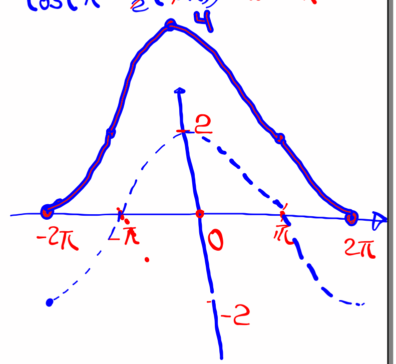
$\cos(\pi - \frac{1}{2}(-2\pi)) = \cos 2\pi = 1$

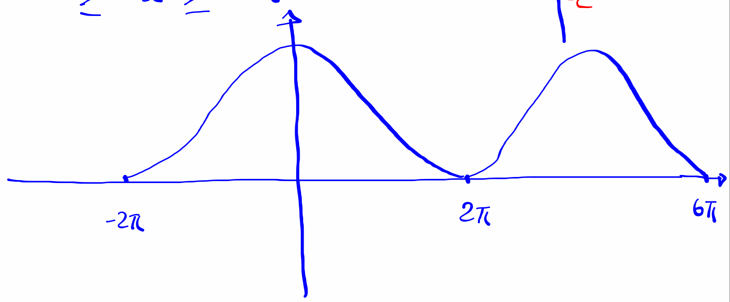
$0 \le \pi - \frac{1}{2}x \le 2\pi$

$0 \le 2\pi - x \le 4\pi$

$-2\pi \le -x \le 2\pi$

$2\pi \ge x \ge -2\pi$





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Graph $y = -2\cos(\pi - \frac{1}{2}x) + 2$

$\pi - \frac{1}{2}x = -\frac{1}{2}x + \pi = -\left(\frac{1}{2}x - \pi\right)$

Recall $\cos(-\alpha) = \cos\alpha$

$\cos\left(\pi - \frac{1}{2}x\right) = \cos\left(-\left(\frac{1}{2}x - \pi\right)\right) = \cos\left(\frac{1}{2}x - \pi\right)$

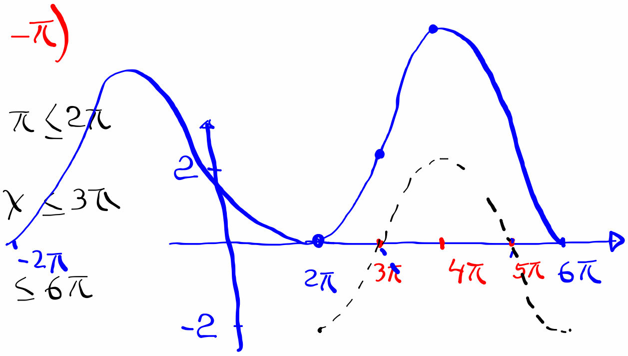
Let's graph

$y = \cos\left(\frac{1}{2}x - \pi\right)$

$0 \le \frac{1}{2}x - \pi \le 2\pi$

$\pi \le \frac{1}{2}x \le 3\pi$

$2\pi \le x \le 6\pi$



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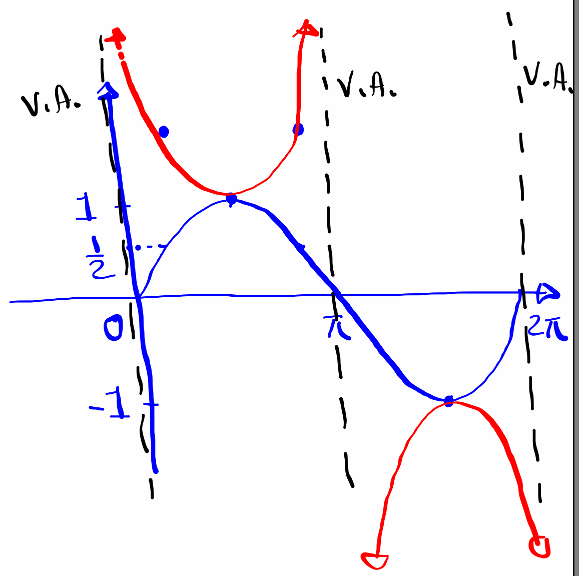
Graph $y = \csc x$

Graph $y = \sin x$

$\frac{1}{.01} = 100$



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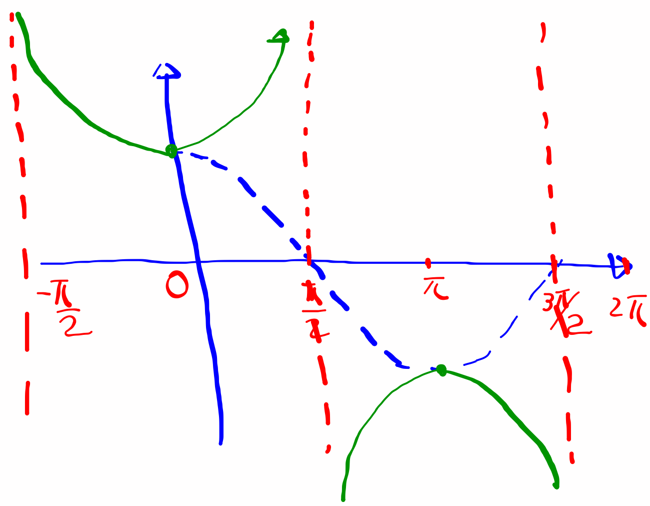
Graph $y = \sec x$

Graph $y = \cos x$



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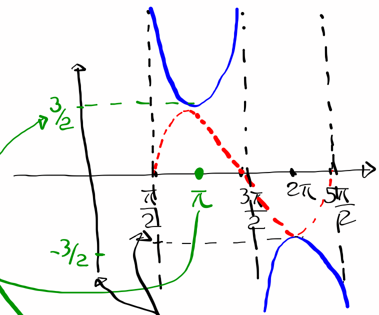
Graph $y = \frac{3}{2}\csc\left(x - \frac{\pi}{2}\right)$

Hint

Graph $y = \frac{3}{2}\sin\left(x - \frac{\pi}{2}\right)$

$0 \le x - \frac{\pi}{2} \le 2\pi$

$\pi/2 \le x \le \frac{5\pi}{2}$



Evaluate $\frac{3}{2}\csc\left(\pi - \frac{\pi}{2}\right)$

$= \frac{3}{2}\csc\left(\frac{\pi}{2}\right) = \frac{3}{2} \cdot 1 = \frac{3}{2}$

Evaluate $\frac{3}{2}\csc\left(2\pi - \frac{\pi}{2}\right) =$

$\frac{3}{2}\csc\left(\frac{3\pi}{2}\right) = \frac{3}{2}(-1) = -\frac{3}{2}$

Basic Graphs are on Exam II

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